

# NAG C Library Chapter Contents

## s – Approximations of Special Functions

s Chapter Introduction

| Routine Name | Mark of Introduction | Purpose                                                                                                |
|--------------|----------------------|--------------------------------------------------------------------------------------------------------|
| s01bac       | 7                    | nag_shifted_log<br>$\ln(1+x)$                                                                          |
| s10aac       | 1                    | nag_tanh<br>Hyperbolic tangent, $\tanh x$                                                              |
| s10abc       | 1                    | nag_sinh<br>Hyperbolic sine, $\sinh x$                                                                 |
| s10acc       | 1                    | nag_cosh<br>Hyperbolic cosine, $\cosh x$                                                               |
| s11aac       | 1                    | nag_arctanh<br>Inverse hyperbolic tangent, $\operatorname{arctanh} x$                                  |
| s11abc       | 1                    | nag_arcsinh<br>Inverse hyperbolic sine, $\operatorname{arcsinh} x$                                     |
| s11acc       | 1                    | nag_arccosh<br>Inverse hyperbolic cosine, $\operatorname{arccosh} x$                                   |
| s13aac       | 1                    | nag_exp_integral<br>Exponential integral $E_1(x)$                                                      |
| s13acc       | 1                    | nag_cos_integral<br>Cosine integral $\operatorname{Ci}(x)$                                             |
| s13adc       | 1                    | nag_sin_integral<br>Sine integral $\operatorname{Si}(x)$                                               |
| s14aac       | 1                    | nag_gamma<br>Gamma function $\Gamma(x)$                                                                |
| s14abc       | 1                    | nag_log_gamma<br>Log Gamma function $\ln(\Gamma(x))$                                                   |
| s14acc       | 7                    | nag_polygamma_fun<br>$\psi(x) - \ln x$                                                                 |
| s14adc       | 7                    | nag_polygamma_deriv<br>Scaled derivatives of $\psi(x)$                                                 |
| s14aec       | 6                    | nag_real_polygamma<br>Derivative of the psi function $\psi(x)$                                         |
| s14afc       | 6                    | nag_complex_polygamma<br>Derivative of the psi function $\psi(z)$                                      |
| s14agc       | 7                    | nag_complex_log_gamma<br>Logarithm of the Gamma function $\ln \Gamma(z)$                               |
| s14bac       | 1                    | nag_incomplete_gamma<br>Incomplete Gamma functions $P(a, x)$ and $Q(a, x)$                             |
| s15abc       | 1                    | nag_cumul_normal<br>Cumulative Normal distribution function $P(x)$                                     |
| s15acc       | 1                    | nag_cumul_normal_complem<br>Complement of cumulative Normal distribution function $Q(x)$               |
| s15adc       | 1                    | nag_erfc<br>Complement of error function $\operatorname{erfc}(x)$                                      |
| s15aec       | 1                    | nag_erf<br>Error function $\operatorname{erf}(x)$                                                      |
| s15afc       | 7                    | nag_dawson<br>Dawson's integral                                                                        |
| s15ddc       | 7                    | nag_complex_erfc<br>Scaled complex complement of error function, $\exp(-z^2) \operatorname{erfc}(-iz)$ |

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| s17acc | 1 | nag_bessel_y0<br>Bessel function $Y_0(x)$                                                                                         |
| s17adc | 1 | nag_bessel_y1<br>Bessel function $Y_1(x)$                                                                                         |
| s17aec | 1 | nag_bessel_j0<br>Bessel function $J_0(x)$                                                                                         |
| s17afc | 1 | nag_bessel_j1<br>Bessel function $J_1(x)$                                                                                         |
| s17agc | 1 | nag_airy_ai<br>Airy function $\text{Ai}(x)$                                                                                       |
| s17ahc | 1 | nag_airy_bi<br>Airy function $\text{Bi}(x)$                                                                                       |
| s17ajc | 1 | nag_airy_ai_deriv<br>Airy function $\text{Ai}'(x)$                                                                                |
| s17akc | 1 | nag_airy_bi_deriv<br>Airy function $\text{Bi}'(x)$                                                                                |
| s17alc | 6 | nag_bessel_zeros<br>Zeros of Bessel functions $J_\alpha(x)$ , $J_\alpha'(x)$ , $Y_\alpha(x)$ or $Y_\alpha'(x)$                    |
| s17dcc | 7 | nag_complex_bessel_y<br>Bessel functions $Y_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$                  |
| s17dec | 7 | nag_complex_bessel_j<br>Bessel functions $J_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$                  |
| s17dgc | 7 | nag_complex_airy_ai<br>Airy functions $\text{Ai}(z)$ and $\text{Ai}'(z)$ , complex $z$                                            |
| s17dhc | 7 | nag_complex_airy_bi<br>Airy functions $\text{Bi}(z)$ and $\text{Bi}'(z)$ , complex $z$                                            |
| s17dlc | 7 | nag_complex_hankel<br>Hankel functions $H_{\nu+a}^{(j)}(z)$ , $j = 1, 2$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$ |
| s18acc | 1 | nag_bessel_k0<br>Modified Bessel function $K_0(x)$                                                                                |
| s18adc | 1 | nag_bessel_k1<br>Modified Bessel function $K_1(x)$                                                                                |
| s18aec | 1 | nag_bessel_i0<br>Modified Bessel function $I_0(x)$                                                                                |
| s18afc | 1 | nag_bessel_i1<br>Modified Bessel function $I_1(x)$                                                                                |
| s18ccc | 2 | nag_bessel_k0_scaled<br>Scaled modified Bessel function $e^x K_0(x)$                                                              |
| s18cdc | 2 | nag_bessel_k1_scaled<br>Scaled modified Bessel function $e^x K_1(x)$                                                              |
| s18cec | 2 | nag_bessel_i0_scaled<br>Scaled modified Bessel function $e^{- x } I_0(x)$                                                         |
| s18cfc | 2 | nag_bessel_i1_scaled<br>Scaled modified Bessel function $e^{- x } I_1(x)$                                                         |
| s18dcc | 7 | nag_complex_bessel_k<br>Modified Bessel functions $K_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$         |
| s18dec | 7 | nag_complex_bessel_i<br>Modified Bessel functions $I_{\nu+a}(z)$ , real $a \geq 0$ , complex $z$ , $\nu = 0, 1, 2, \dots$         |
| s18ecc | 6 | nag_bessel_i_nu_scaled<br>Scaled modified Bessel function $e^{-x} I_{\nu/4}(x)$                                                   |
| s18edc | 6 | nag_bessel_k_nu_scaled<br>Scaled modified Bessel function $e^x K_{\nu/4}(x)$                                                      |
| s18eec | 6 | nag_bessel_i_nu<br>Modified Bessel function $I_{\nu/4}(x)$                                                                        |
| s18efc | 6 | nag_bessel_k_nu<br>Modified Bessel function $K_{\nu/4}(x)$                                                                        |

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| s18egc | 6 | nag_bessel_k_alpha<br>Modified Bessel functions $K_{\alpha+n}(x)$ for real $x > 0$ , selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$                          |
| s18ehc | 6 | nag_bessel_k_alpha_scaled<br>Scaled modified Bessel functions $K_{\alpha+n}(x)$ for real $x > 0$ , selected values of $\alpha \geq 0$ and $n = 0, 1, \dots, N$            |
| s18ejc | 6 | nag_bessel_i_alpha<br>Modified Bessel functions $I_{\alpha+n-1}(x)$ or $I_{\alpha-n+1}(x)$ for real $x \neq 0$ , non-negative $\alpha < 1$ and $n = 1, 2, \dots,  N  + 1$ |
| s18ekc | 6 | nag_bessel_j_alpha<br>Bessel functions $J_{\alpha+n-1}(x)$ or $J_{\alpha-n+1}(x)$ for real $x \neq 0$ , non-negative $\alpha < 1$ and $n = 1, 2, \dots,  N  + 1$          |
| s18gkc | 7 | nag_complex_bessel_j_seq<br>Bessel function of the 1st kind $J_{\alpha \pm n}(z)$                                                                                         |
| s19aac | 1 | nag_kelvin_ber<br>Kelvin function $\text{ber } x$                                                                                                                         |
| s19abc | 1 | nag_kelvin_bei<br>Kelvin function $\text{bei } x$                                                                                                                         |
| s19acc | 1 | nag_kelvin_ker<br>Kelvin function $\text{ker } x$                                                                                                                         |
| s19adc | 1 | nag_kelvin_kei<br>Kelvin function $\text{kei } x$                                                                                                                         |
| s20acc | 1 | nag_fresnel_s<br>Fresnel integral $S(x)$                                                                                                                                  |
| s20adc | 1 | nag_fresnel_c<br>Fresnel integral $C(x)$                                                                                                                                  |
| s21bac | 1 | nag_elliptic_integral_rc<br>Degenerate symmetrised elliptic integral of 1st kind $R_C(x, y)$                                                                              |
| s21bbc | 1 | nag_elliptic_integral_rf<br>Symmetrised elliptic integral of 1st kind $R_F(x, y, z)$                                                                                      |
| s21bcc | 1 | nag_elliptic_integral_rd<br>Symmetrised elliptic integral of 2nd kind $R_D(x, y, z)$                                                                                      |
| s21bdc | 1 | nag_elliptic_integral_rj<br>Symmetrised elliptic integral of 3rd kind $R_J(x, y, z, r)$                                                                                   |
| s21cac | 7 | nag_real_jacobian_elliptic<br>Jacobian elliptic functions $\text{sn}$ , $\text{cn}$ and $\text{dn}$ of real argument                                                      |
| s21cbc | 6 | nag_jacobian_elliptic<br>Jacobian elliptic functions $\text{sn}$ , $\text{cn}$ and $\text{dn}$ of complex argument                                                        |
| s21ccc | 6 | nag_jacobian_theta<br>Jacobian theta functions with real arguments                                                                                                        |
| s21dac | 6 | nag_elliptic_integral_f<br>Elliptic integrals of the second kind with complex arguments                                                                                   |
| s22aac | 6 | nag_legendre_p<br>Legendre and associated Legendre functions of the first kind with real arguments                                                                        |

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